DENSITY DISTRIBUTION IN PULSED GAS JETS EFFUSING INTO A RAREFIED SPACE

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The results of measurements of the density in pulsed jets with large pressure mismatch are presented. Techniques for obtaining information about the local values of the density, based on measurements of the integral attenuation of a probe electron beam under specific conditions of nonstationary jet flow, are discussed. In estimating the consequences of pulsed directed emissions, very simplified flow schemes are, as a rule, studied. It is suggested that the flow field can be estimated either from relations and results of calculations for stationary jets or models of flow from a spatially symmetric, instantaneously switched on, stationary source. Both approaches are approximate and must be refined when more reliable information about the character of the spatial distribution of the flow parameters as a function of time is required.

In [1-3] the results of an experimental study of pulsed gas jets, formed with starting pressure mismatch $(P_0/P_\infty)\cdot 10^8$ and pressure in the background space $P_\infty \sim 10^{-5}$ mm Hz, are given. The density field of nitrogen and argon jets in the process of flow development are constructed based on measurements of the integral absorption of an electron beam. Data on the motion of the front of the jet and the characteristics of how the background space is filled with the gas jet formed were obtained.

By measuring the integral attenuation of an electron beam it is possible to obtain information about the maximum sensitivity of the diagnostics apparatus under conditions when other methods of local diagnostics of rarefied flows become problematic. At the same time, the determination of the local value of the density based on data on integral absorption, which, when the geometry of the flow is known a priori (in view of its symmetry), reduces to the solution of Abel's equation [4-6], is a difficult problem.

Analysis of the character of the primary experimental data and the results of previous calculations showed that the distribution of the density in the pulsed jets studied can be described analytically based on the experimental data.

Figure 1a shows typical oscillograms of the absorption of an electron beam (the x axis is oriented along the jet away from the nozzle cutoff and the y axis is oriented perpendicular to the axis of the jet, Φ marks the onset of absorption, and c marks the attainment of the quasistationary state). In the experiments only the variable component of the beam current was recorded; this made it possible to increase substantially the accuracy of the measurements. Figure 1b shows the probing scheme.

Since in the experiments the axisymmetric flow is probed along a chord in order to determine the density, the relation between the density at a point with coordinates (y, ξ) in a Cartesian coordinate system (or with the coordinate r in the polar coordinate system; see Fig. 1b) and the value of the absorption along the chord y_i is given by

$$F(y) = 2\int_0^{\sqrt{R^2-y^2}} \rho\left(\sqrt{y^2+\xi^2}\right) d\xi. \tag{1}$$

The substitution of variables $r^2 - y^2 = \xi^2$ reduces the relation (1) to Abel's equation

$$F(y) = 2\int_y^R \frac{\rho(r)\, r\, dr}{\sqrt{r^2-y^2}} \tag{2}$$

(R is the radius of the boundary of the jet).

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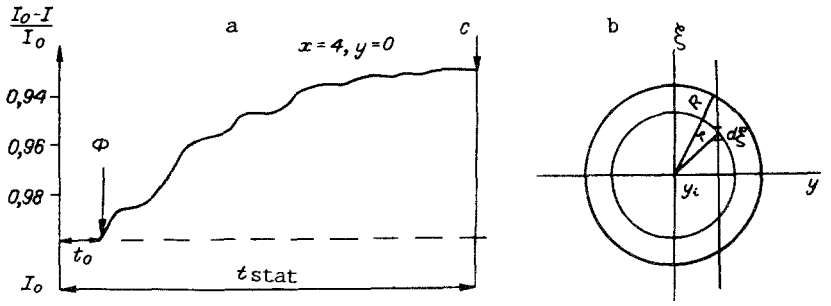

Fig. 1

The function

$$\rho(r)=\frac{1}{\pi}\left\{\frac{F(R)}{\sqrt{R^2-r^2}}-\int_r^R\frac{\frac{dF}{dy}dy}{\sqrt{y^2-r^2}}\right\} \tag{3}$$

is the solution of Eq. (2) [7]. The character of the arrangement of the measured values of the absorption gives a basis for approximating the measured values by a two-parameter function of the form $F(y) = ae^{-by^2}$ for $y > R$. In accordance with the physical meaning of the problem, for $y \geq R$ we have $F(y) = 0$, if R is chosen so that the absorption of the probe beam for $y \geq R$ is equal to the absorption in the background gas, i.e., the measured signal $F(y) = \begin{cases} ae^{-by^2}, & y < R \\ 0, & y \geq 0 \end{cases}$ in Eq. (3) gives

$$\rho(r)=\frac{a\sqrt{b}}{\pi}\Phi\left(\sqrt{2b(R^2-r^2)}\right)e^{-br^2}, \tag{4}$$

where Φ is the error function $\Phi(x) = \frac{2}{\sqrt{2\pi}}\int_0^x e^{-t^2/2}dt$ [$\Phi(x) > 0.95$, if $x > 2$]. In our case

$$\Phi(\sqrt{2b(R^2-r^2)})\cong 1,\ b=(R^2-r^2)>2. \tag{5}$$

We now introduce the maximum sensitivity of the measuring apparatus $\Delta = ae^{-br^2}\max$ and define in terms of it

$$r_{\max}^2=\frac{1}{b}\ln\frac{a}{\Delta}. \tag{6}$$

If $R^2 > 1/b \ln(a/\Delta) + (2/b)$, which follows from Eqs. (5) and (6), based on Eq. (4) we obtain

$$\rho(r)=\frac{a\sqrt{b}}{\sqrt{\pi}}e^{-br^2}. \tag{7}$$

If $F(y) = ae^{-by^2}$, $0 \leq y < \infty$, is used as a smoothing function, i.e., the requirement $F(y) = 0$ for $y \geq R$ is ignored, then Eq. (7) is an exact solution of Eq. (3); this can be verified by direct substitution of Eq. (7) into Eq. (3). The nonzero values of the density given by such an approximation for any argument are justified by the specific nature of the experimental results being analyzed.

The real density for $y > R$ is actually not equal to zero. Apart from the background gas, the effusing gas is always present behind the formal boundary of the jet because molecules are separated in the process of effusion [8] and because the boundary of the jet is not sharp.

The approximations examined above have certain "flaws" of different mathematical nature, but they give solutions of the same form, so that any approximation studied is acceptable. Satisfaction of the conditions (5) will be checked after the parameter b is determined. To determine a and b we will use the method of least squares (MLS), according to which these parameters can be found based on the set of values of the absorption $(F_1 \ldots F_n)$ by solving the system of equations

$$\frac{\partial S}{\partial a}=0,\quad \frac{\partial S}{\partial b}=0\quad\left(S=\sum_{i=1}^{n}\left(ae^{-by_i^2}-F_i\right)^2\right). \tag{8}$$

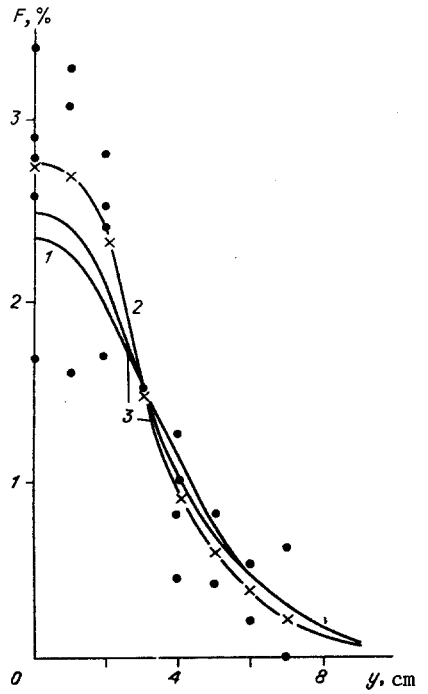

Fig. 2

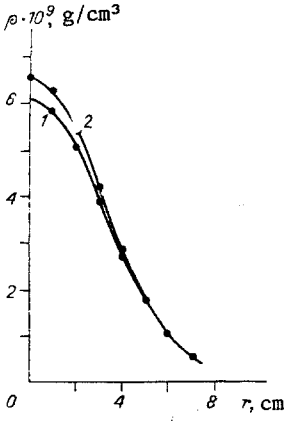

Fig. 3

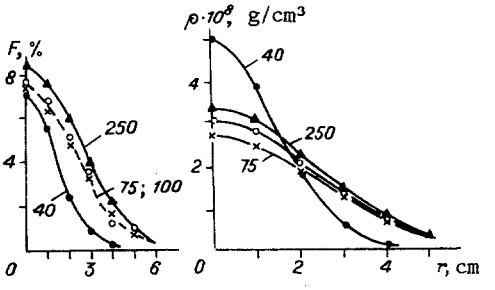

Fig. 4

From the starting equations, after substitution and reduction of the system to one equation for one unknown, we write

$$\sum_{i=1}^{n} y_i^2 e^{-2by_i^2} \sum_{i=1}^{n} F_i e^{-by_i^2} = \sum_{i=1}^{n} e^{-2by_i^2} \sum_{i=1}^{n} F_i y_i^2 e^{-by_i^2}, \tag{9}$$

where $a = \left(\sum_{i=1}^{n} F_i e^{-by_i^2}\right) \Big/ \left(\sum_{i=1}^{n} e^{-2by_i^2}\right)$. It is obvious from Eq. (9) that different measurements F_i have a different weight. As the argument y_i increases the weight decreases, so that for sufficiently large values of the argument the measurements do not affect the result and are therefore useless.

The theory of the method of least squares recommends, for purposes of simplifying the solution and based on some fundamental considerations, that the system of equations be reduced by means of appropriate transformations to a linear form [5]. Once the system (8) is linearized, we find the coefficients a and b by taking the logarithm and performing the required transformations. From Eq. (7) (taking into account the physical characteristics of the gas under study) we obtain the value of the density as a function of the radial coordinate.

For all regimes realized in the experiment, the values of b fall into the range $0.05 \leq b \leq 0.2$ and the ratio a/Δ falls into the range $30 \leq a/\Delta \leq 100$, in accordance with which R = 7.75 cm. In the experiments the absorption was recorded reliably for $r_{max} \leq 7.5$; for larger values the absorption was equal to zero. Therefore, an approximation of the form $F(y) = ae^{-by^2}$ is well-founded in the entire range $0 \leq y < \infty$. This approximation agrees with the possibilities of the measurement method and its accuracy is at least as good as that of the measurements, which is determined by the sensitivity of the apparatus. Obviously, the methods of electron-beam diagnostics under conditions of pulsed jet flows will be more effective if multiaspect and multibeam probing is employed; in this case, the possibility of using an analytical approximation greatly simplifies the processing of the experimental data and makes it possible to automate the processing.

We present below specific examples of the analysis of experiments and comparison of the experimental results with the results of numerical calculations using a graphical approximation of absorption. Figure 2 shows the beam-absorption data obtained in one section of an argon jet at an obviously nonstationary stage of the flow. The measurements were performed along seven chords; the total number of points is equal to 24. For the analysis we chose the section in which irregularity in the measured quantities is clearly manifested

because the measurements were made far enough away from the nozzle cutoff and because the time from the start of effusion was comparatively short. The curve 1 shows the approximation of the experimental values directly; the curve 2 is drawn through the arithmetic mean of the experimental values; and, the curve 3 is an approximation curve, constructed based on the arithmetic-mean values. The fact that the curves 1 and 3 do not coincide reflects the effect of the different weight of the points in the regions of the flow near the axis and at the periphery. In Fig. 2 one can see that the approximation relations introduce a correction to the arithmetic mean values, determined differently for each y_i. The values at the maximum are too low and beyond the point of inflection they are too high compared with the arithmetic mean values. The maximum difference in the density is 7% (Fig. 3). Comparing the results obtained using the analytical and geometric methods of approximating the experimental data shows that in the second case the density drops more rapidly at the periphery. This occurs because the computational method presupposes that a boundary is introduced on which the absorption signal is equal to zero: There is no effusing gas beyond $r \geq R$. In reality, however, the boundary of the jet is not sharp, both because of diffusion and interpenetration of the gases of the jet and of the background space as well as, as pointed out above, owing to the separation of the molecules of the effusing gas in the zone near the front. The asymptotic decay of the absorption described by $F = a\ e^{-br^2}$ best corresponds to the physical picture of the process.

Figure 4 illustrates another experiment in which an analytical smoothing function was constructed and the corresponding density distribution was determined. Here data for a nitrogen jet in the section x = 2 cm are compared; the measurements were performed 40, 75, 100, and more than 250 μsec after effusion starts. Curves of the density distribution were constructed from the smoothed functions approximating the values of the absorption. From 75 to 100 and 250 μsec the density in the region near the axis increases smoothly. At the periphery of the jet ($r > 5$) the absorption is small; for all times it is close to the limit of sensitivity and the density (for $r > 5$) is virtually constant. For 40 μsec the density on the axis is 1.5 times higher than at later times; the density drops more rapidly at the periphery. It can be conjectured that the demonstrated character of the change in the density is connected with the formation of vortex structures, analogous to those observed at the initial stage of development of pulsed jets, effusing into a space with a higher counterpressure [2].

Comparing the absolute and relative values of the density, found in experiments for quasistationary flows of different gases, reveals a tendency manifested in model calculations of jets effusing into a vacuum: the argon jet is narrower and has a longer range than the nitrogen jet.

Comparing the methods used previously [1-3] and the methods presented in this paper for processing the experimental data, it is useful to note that the evolution of the density field, obtained by analyzing the measurements of the integral attenuation of a beam, is qualitatively the same in both cases. The quantitative discrepancies are largest where the spread in the measurements is maximum; the maximum spread is unique. The reliability of the density values found from experiments on integral absorption cannot be predicted a priori independently of the method used to extract this information. By regularizing the measurements geometrically, we obtain information directly and we avoid the mathematically difficult step of fitting a function which gives the best description of the actual coordinate dependence of the absorption. But the existence of natural irregularities and the random spread in the measurements unavoidably introduce distortions into the reconstructed density function $\rho(r)$, and since $\rho(r)$ is unknown, both the scale and the character of the distortions are unpredictable. Solving Abel's equation numerically does not save the situation, although in so doing the unaveraged, practically primary results of the measurements are employed.

Analytical approximation is preferable because it is simpler and it makes it possible to automate the processing of the measurements directly in the course of the experiment. Analytically described results are more convenient to analyze for further scientifc and applied use.

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SYMMETRIZATION OF THE EQUATIONS OF DYNAMICS OF A CAPILLARY LIQUID

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A model of the motion of a nonisothermal capillary liquid in the presence of surfactants at low concentration is described. A symmetric form of the equations in the intensive variables temperature–chemical potential is derived based on fundamental principles of thermodynamics.

From the mechanical standpoint, capillary forces are internal forces and the problem of determining them is a problem in the rheology of a multiphase medium. The main difference between such a continuous medium and classical liquids is that in the phase mixing layer the stress tensor is anisotropic (Pascal's law is not satisfied). The thickness of the mixing layer is equal to several intermolecular distances and it remains virtually constant during the motion of the liquid. For this reason, it is natural to model this layer as an interphase surface Γ with distributed excess thermodynamic quantities ε (internal energy), η (entropy), and γ (concentration of surfactant molecules), making the assumption that the layer is an open thermodynamic system in contact with volume phases – a reservoir of heat and surfactant particles. This approach of Gibbs makes it possible to circumvent the complicated question of the structure and thickness of the phase mixing layer and to use more efficient thermodynamic methods [1].

The postulated principle of local thermodynamic equilibrium leads to the condition that the intensive parameters θ (absolute temperature) and ζ (the chemical potential of the surfactant), which are characteristics of the reservoir, are continuous. This condition makes it possible to extend the relations of equilibrium thermodynamics or thermostatics to nonstationary processes involved in the dynamics of an interphase boundary. Here the principle of minimum entropy production is very important. This principle must be used in order to construct the correct relation between the heat flux $\mathbf{q}$ and the flux of surfactant molecules $\mathbf{j}$ and the gradients of θ and ζ [2].

1. Thermodynamics of the Interphase Boundary. Consider a closed thermodynamic system consisting of two phases separated by a uniform interphase surface of area A. Let E be the total energy of the system, N be the number of surfactant molecules, $S(E, N)$ be the entropy of the volume phases, and $A\eta(\varepsilon, \gamma)$ be the excess entropy of the surface. According to the principle of maximum total entropy $S(E - A\varepsilon, N - A\gamma) + A\eta(\varepsilon, \gamma) \to \max$ in the approximation $A\varepsilon \ll E$, $A\gamma \ll N$ (the reservoir is much larger than the system in contact with it)

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